

Soft Matter Problem set 4 – Interfaces and surfactants

Answers

Problem 13

Use the molecular view of interfacial tension ($\gamma \approx u/2a^2$) to explain which of the following interfaces have a higher interfacial tension.

- At the water-ice interface, hydrogen bonds can still be formed, although fewer than in bulk water. Air-water therefore has a higher interfacial tension.
- b. At the oil-water interface, Van der Waals interactions are still present between water molecules and oil molecules, whereas the number of molecules in air is so small by comparison, that the interactions between water and air are negligible. Air-water therefore has a higher interfacial tension.
- c) If we assume a typical energy scale of about $1 k_B T$ per colloidal particle as an interparticle interaction energy in the colloidal liquid state, regardless of their size, the interfacial tension is proportional to the energy scale per unit area, so the smallest particles will have highest interfacial tension. In general, this analysis suggests that colloidal fluids can have ultralow interfacial tensions, as the size of colloidal particles is typically 3-4 orders of magnitude larger than the molecular size. This is indeed what has been found experimentally, see for example: Aarts, Dullens, Lekkerkerker, New Journal of Physics 2005.

Problem 14

a)

$$\Delta p = p_A - p_0 = -\frac{2\gamma \cos \theta}{R}$$

(Two radii of curvature are the same for a cylindrical capillary.)

b)

$$p_B = p_C = p_0$$

c)

$$p_B - p_A = \frac{F_g}{A} = \frac{m_{\text{water}} g}{A} = \frac{\rho g V_{\text{water}}}{A} = \rho g h$$

d)

$$\rho g h = \frac{2\gamma \cos \theta}{R} \quad h = \frac{2\gamma \cos \theta}{\rho g R}$$

For $\gamma = 72 \text{ mN/m}$, $\theta = 0^\circ$, $\rho = 1000 \text{ kg/m}^3$, $R = 147 \text{ nm}$.

Problem 15

- A heterogeneous surface (different wettabilities, because of different surface energy or different roughness)

- b) Laplace pressure top: $\Delta p_L = \frac{2\gamma \cos \theta_1}{R}$ Laplace pressure bottom: $\Delta p_L = \frac{2\gamma \cos \theta_2}{R}$ Hydrostatic pressure difference: $\Delta p_h = \rho g h$ Stable if:

$$\frac{2\gamma \cos \theta_1}{R} > \frac{2\gamma \cos \theta_2}{R} + \rho g h$$

$$\cos \theta_1 - \cos \theta_2 > \frac{\rho g h R}{2\gamma}$$

Problem 16

a)

$$x_n = x_1 = x$$

$$x = n x^n e^{n\Delta\mu/k_B T}$$

$$\frac{e^{-n\Delta\mu/k_B T}}{n} = x^{n-1}$$

$$x = \left(\frac{e^{-n\Delta\mu/k_B T}}{n} \right)^{1-n} = \left(\frac{K_n}{n} \right)^{1-n}$$

b)

$$\log x_n = \log n + n \log x_1 + n\Delta\mu/k_B T$$

Slope = n

Intercept = $\log n + n\Delta\mu/k_B T$

- c) Volume of a micelle = $V_{\text{micelle}} = \frac{4}{3}\pi\ell^3 \approx 20.9 \text{ nm}^3$

$$N = \frac{V_{\text{micelle}}}{v} = 58$$

- d) Charge screening by salt ions decreases the repulsion between the charged head groups of SDS, so the CMC will go down. N will increase because a_0 decreases, and therefore v decreases.